The Effects of Leniency on Illegal Transactions:
How (Not) to Fight Corruption*

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Abstract

We study the consequences of leniency – reduced legal sanctions for wrongdoers who spontaneously self-report to law enforcers – on corruption, drug dealing, and other forms of sequential, bilateral, illegal trade. We find that when not properly designed, leniency may be highly counterproductive. In reality leniency is typically “moderate,” in the sense of only reducing, or at best cancelling the sanctions for the self-reporting party. We find that moderate leniency may facilitate the enforcement of long-term illegal trade relations, and may even provide an effective enforcement mechanism for occasional (one-shot) illegal transactions, which would not be enforceable otherwise.

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The son said to him, “Father, I have sinned against heaven and against you. I am no longer worthy to be called your son”. But the father said to his servants, “Quick! Bring the best robe and put it on him. Put a ring on his finger and sandals on his feet. Bring the fattened calf and kill it. Let’s have a feast and celebrate”. (Luke 15, 21-23)

1 Introduction

The economic and social costs of illegal transactions are enormous. The direct and indirect costs of illegal trade in drugs, arms, and toxic waste are so obvious and huge that need not be emphasized here. In these last years, criminal organizations also rediscovered the trafficking of human beings (kids and girls for sexual exploitation, refugees, illegal immigrants to feed sweat-shops\(^1\)). Even the economists’ traditional benevolence towards corruption, seen as a way to overcome excessive regulation, has been heavily questioned by recent studies showing that corruption may reduce investment, financial development, and growth.\(^2\)

Illegal transactions normally suffer of “enforcement problems,” since to constrain each other’s opportunism transacting parties cannot rely upon explicit contracts enforced by the legal system.\(^3\) Unless the exchange – say, between a firm’s bribe and a bureaucrat’s favor

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\(^1\)For an introductory and depressing overview of the revival of this business, also in Europe and the US, the reader can check http://www.unhcr.ch/evaluate/reports/traffick.pdf; http://www.savethechildren.it/pdf/child Trafficking_Alabania.pdf; http://www.catwinternational.org; and http://207.153.255.161/ecpat1/index2.htm.

\(^2\)Bardhan (1997) offers an overview of the early literature; hundreds of economic papers have been published since then. Mauro’s (1995) pioneering analysis estimates that a one standard deviation improvement in a country corruption index is associated with an increase in the investment rate by about 3 percent of GDP. Recent work on transition economies places corruption at the heart of their poor post-privatization performance (see e.g. Boycko et al. 1995; McMillan et al. 1999; Black et al. 1999; and particularly Hellman et al. 2000).

\(^3\)We wrote “normally” because in some countries the Mafia partially solves these problems by taxing illegal transactions and offering “enforcement services” (see e.g. Gambetta, 1993; Gambetta and Reuter,
can be perfectly simultaneous, illegal transactions need be repeated frequently enough. Then self-enforcing exchange relations can be sustained, as parties find it convenient to stick to the agreement to maintain their reputation of “honest criminals,” avoid retaliations/punishments, and realize the expected future flow of gains from illegal trade.4

Since illegal transactions involve at least two parties and require a certain degree of internal trust, one way law enforcement agencies fight them is by shaping private incentives to play one party against the other(s); that is, by ensuring that agents involved in illegal transactions find themselves in a situation as close as possible to a Prisoner’s Dilemma.5 Law enforcers do this by awarding leniency – typically a reduction or cancellation of legal sanctions (accompanied by protection from retaliation and related benefits) – to wrongdoers that report their illegal behavior and sufficient information to convict “the rest of the gang.” Awarding leniency to wrongdoers who cooperate is probably the most important instrument in the hands of law enforcers to elicit information on organized crime. Formal and informal exchanges of leniency against information/collaboration are a normal feature of law enforcement in most world countries. In particular, they have been extensively and quite successfully used (and misused) in the US and Italy to fight Sicilian Mafia, and are routinely used (and misused) in the US to fight drug dealing and related crimes.6 From

1995)

4 This feature is common to most kinds of organized crime (e.g. Polo 1995; Garoupa 1999). Nevertheless, deterring illegal trade, and particularly corruption is usually quite problematic. This is because where corruption and illegal trade is more widespread the law enforcing agencies in charge of monitoring and sanctioning illegal behavior are also typically inefficient and corrupt (law enforcement agencies are often the first part of the administration to get corrupted, and economists spent already some effort to understand how the incentives of these agencies should be structured to minimize corruption; see e.g. Mookherjee and Png 1995; Polinsky and Shavell 1999).

5 Raising sanctions against participants is the other, more standard way of deterring illegal transactions (and crime in general; see Becker 1968). Though, the literature on the optimal enforcement of law has identified several reasons why raising sanction above certain thresholds can be counterproductive (see e.g. Polinsky and Shavell 2000).

6 The misuse occurs when prosecutors and courts rely exclusively (or mainly) upon a testimony obtained in exchange for leniency. A useful introduction to this incredible practice is at http://www.pbs.org/wgbh/pages/frontline/shows/snitch/. Throughout the paper we will assume that the
a theoretical point of view, the Prisoner’s Dilemma game itself is perhaps the best known model of leniency: the sanctions for a prisoner that unilaterally confesses his crime are reduced to induce him to confess and prove guilty his former partner(s).

The Prisoner’s Dilemma refers to a situation in which the crime took place, the joint law violators are already under investigation, and leniency aims at eliciting their information to reduce the costs of prosecution and maximize the probability of proving them guilty. Leniency is also advocated and implemented as a way to directly deter crime and reduce investigation costs by providing incentives for undetected law violators to spontaneously self-report. This paper focuses on the deterrence effects of this kind of leniency, awarded to agents involved in illegal transactions who spontaneously report their behavior when they are not under any sort of investigation or prosecution. We analyze how the opportunity to obtain leniency by spontaneously self-reporting affects the enforceability of occasional and repeated illegal exchanges.

Focusing on isolated wrongdoers committing individual crimes, in the tradition of Becker’s (1968) seminal contribution, economists already highlighted a number of important benefits that awarding leniency to agents that spontaneously self-report may bring about. party applying for leniency must report “hard information” against his partners to obtain it, and that his testimony is not required nor admitted.

7 Leniency awarded during prosecution in the context of antitrust law enforcement is the focus of a recent paper by Motta and Polo (1999) pointing out, among other things, that its effects on deterrence are a priori ambiguous: on the one hand, leniency increases the probability of being sanctioned by increasing chances that prosecution is successful; on the other hand, leniency reduces the sanctions for wrongdoers that self-report, thereby reducing ex ante expected sanctions.

8 In most legal systems a criminal that spontaneously goes to prosecutors and confesses his crime (or sends his lawyer first to strike a deal) is eligible for the strongest possible leniency. In Italy, after the corruption scandals of the early 90s, it has been proposed from many sides to cancel legal sanctions against firms that spontaneously report having bribed bureaucrats to increase the riskiness of corruption for the lasts. In some Middle-East countries, like Egypt, this form of leniency has been codified and implemented for several years in the attempt to induce firms to denounce corrupt bureaucrats.

9 Note that from a narrowly economic point of view deterrence is all what matters, it is the ultimate objective of law enforcement. If one disregards moral concerns (including justice-related psychological effects on victims), were prosecution to have no deterrence effects it would be a pure deadweight cost for society.
The path-breaking models of Kaplow and Shavell (1994) and Malik (1993) showed that offering leniency to wrongdoers that spontaneously self-report generally lowers law enforcement costs by reducing the number of wrongdoers to be detected; Kaplow and Shavell (1994) also showed that it may increase welfare by reducing the overall risk agents bear; and Innes (1999a,b) highlighted the value of the early remediation of damages from crime self-reporting entails. On forms of crime involving more than one party, such as illegal transactions, leniency for self-reporting wrongdoers can have a particularly strong deterrence effect, as the different parties can be played against each other. Spagnolo (2000) showed recently that – when optimally designed – leniency for undetected wrongdoers that spontaneously self-report can in principle costlessly deter all illegal relations enforced by reputational considerations, making the investigation activity itself redundant. To have such pervasive effects leniency should not only reduce sanctions, it should reward wrongdoers that spontaneously report and turn in their partners. Instead, for a number of reasons, in reality leniency is mostly advocated and implemented only up to the cancellation of the legal sanctions for the reporting party.¹⁰

In this paper we model bilateral, sequential, illegal transactions, occasional and repeated, where both the timing/sequence of the exchange and the distribution of illegal gains are endogenously chosen by the illegal partners to facilitate the enforcement of the transactions, given the legal framework. We characterize the effects of leniency under all parameter configurations, and find that the moderate forms of leniency typically implemented in reality can indeed be highly counterproductive.

We find that moderate leniency may facilitate long-term illegal trade relations by making punishments stronger and by reducing agents’ gains from cheating on illegal partners. Mod-

¹⁰Of course, there may be drawbacks in rewarding law violators that spontaneously self-report. Abstracting from moral considerations (totally unwarranted according to the Bible), one potential drawback is that it gives agents incentives to distort/fabricate information. But this drawback can be addressed directly – by restricting eligibility to agents reporting “hard” (easily verifiable) information, not letting agents that obtained leniency testify, and substantially raising sanctions for misreporting – rather than indirectly, by giving up the potentially large benefits of “high powered” leniency.
erate leniency gives cheated upon parties incentives to self-report during the punishment phase that follows unilateral defections. This reinforces the threat that disciplines the illegal trade relation, augmenting it of the entire cost of legal sanctions. The stronger punishment forces wrongdoers that cheat on the illegal agreement to simultaneously self-report (to avoid others reporting on them during the punishment phase) and face reduced legal sanctions, which in turn diminish gains from cheating on partners.

More strikingly, we find that moderate leniency may provide an effective enforcement mechanism for occasional, sequential illegal transactions that would not be enforceable in its absence—nor in the absence of law enforcement altogether! Occasional sequential transactions are difficult to enforce as the party that delivers first has no credible threat available to induce the other party to stick to its promise. For example, in a one-shot firm-bureaucrat corrupt transaction, suppose the firm pays the bribe first. The bureaucrat can then simply not deliver the promised illegal favor. The firm might threaten to retaliate by reporting to the police, but such threat would be empty. The firm would eventually not report, since if it would, it would also face the full legal sanctions against corruption. The converse happens if the illegal favor is delivered by the bureaucrat before the payment of the bribe. Moderate leniency alters this situation by providing would-be corrupt parties with the credible threat needed to enforce the deal. Consider leniency that reduces legal sanctions for firms that spontaneously reports having bribed a bureaucrat to a monetary fine smaller than the paid bribe. Then, if the bureaucrat accepts the bribe but does not deliver the promised illegal favor, the firm has incentives to report, help law-enforcers to prosecute the bureaucrat, recover the bribe and pay the fine. Knowing this, the bureaucrat delivers the illegal favor. If the bureaucrat also delivers, then none of the parties has incentives to report, since by doing it they would lose gains from (illegal) trade. By making the threat of reporting in case of non-compliance credible, leniency enforces an occasional corrupt deal that would not be feasible otherwise. And of course, when the one-shot illegal transaction becomes enforceable, the long-term illegal relation generated by the repetition of the one-shot transaction also becomes enforceable, independent of how often it is repeated and of the intertemporal discount factor.
Finally, we complete the characterization of the effects of leniency by identifying the parameter configurations that makes it effective in terms of deterrence of self-enforcing sequential illegal relations, and confirm Spagnolo’s (2000a) result: sufficiently generous leniency (rewards for self-reporting) could – in principle – costlessly deter all illegal transactions.

Section 2 describes the model(s); Section 3 analyzes one-shot illegal transactions; Section 4 analyzes repeated illegal relations; Section 5 presents a unified, general characterization of the effects of leniency; and Section 6 concludes.

2 The model

2.1 Set up

For concreteness we phrase the model as a corrupt exchange between an entrepreneur and a bureaucrat, but everything could be restated in terms of a transaction between a drug producer and a drug retailer, or of any other illegal transaction. Therefore, the main results will be stated in more general terms.

There are two agents, a bureaucrat (B) and an entrepreneur (E). The entrepreneur has an investment opportunity with net preset value $v > 0$ that can be realized only if the bureaucrat performs an action $a$, illegal or contrary to his duties. This action entails a private cost $c$, and the bureaucrat may require compensation to perform it, a bribe $b$ with $c \leq b \leq v$.\footnote{In the “drug deal” interpretation of the model, $c$ would be the investment cost for the drug producer, $v$ the value of the drug for the retailer, $v - c$ gains from trade and $b$ the agreed price.}

We define corruption as any agreement between the bureaucrat and the entrepreneur according to which the former should do $a$ and the latter should pay $b$. Corruption takes place when at least one party implements the terms of the agreement ($E$ pays $b$, or $B$ does $a$).

To simplify exposition we assume that law enforcers have no resources, so they are not able to (investigate and) detect corruption unless one of the players reports information
on the agreement.\footnote{One could easily introduce a strictly positive probability of detection $0 < p < 1$ when nobody reports, and replace the value of the project $v$ with its expected value net of fines $(1 - p)v - p(F_E + b)$ in the entrepreneur’s payoff, and the bribe $b$ with net expected bribe $(1 - p)b - pF_B$ in the payoff of the bureaucrat. Non-monetary sanctions could also be introduced. Such richer but more cumbersome formulations would perhaps improve the realism of the model, but would not affect its fundamental results.} If this happens, corruption is proved with certainty and players are sanctioned. The normal sanction for an agent $i$ proved guilty of corruption, $S_i$, consists in the confiscation of any illegal gain or payment ($v$, $b$), plus a fine $F_i$, with $i = B, E$ and $F_i > 0$; that is, $S_i = g_i + F_i$, where $g_B \in \{0, b\}$ and $g_E \in \{0, v\}$, depending on the stage of the exchange at which agents are discovered and sanctioned.

Leniency consists in reducing the sanction for a wrongdoer when this spontaneously denounces the illegal transaction and provides evidence sufficient to convict the other offender. We let $RF_i$, $i = B, E$, denote the reduced monetary fine for a player $i$ that obtains leniency, with $RF_i \leq F_i$; and let $RS_i$ denote the overall reduced sanction, with $RS_i = RF_i + g_i$. Since we are taking $F_i$ for given, a “leniency program” (or leniency policy; “LP” from now on) is completely defined by the fine reductions for the two players $(RF_E, RF_B)$. Of course, when $RF_i = F_i$ ($RS_i = S_i$) for all $i$ we are in the case of “no leniency.”

LPs may well establish positive transfers (negative additional fines) $-RF_i > 0$, as when they establish the restitution of part or all of the paid bribe to a briber that spontaneously turns in the bribee. Note, however, that even when the additional transfer is negative the overall reduced sanction $RS_i = RF_i + g_i$ remains positive as long as $-RF_i < g_i$. A leniency program gives a positive reward to the reporting agent only when $-RF_i > g_i$, so that $-RS_i = -RF_i - g_i > 0$.

Finally, we let $L$ denote the set of all available LPs and $l \in L$ a generic LP.

\subsection*{2.2 Timings}

Illegal exchanges are carried out over time. We model the execution of one corrupt transaction as a sequential game that can take two alternative timings.

In the first timing, after players agree on $a$ and $b$ the bureaucrat begins by performing his action $a$, then the entrepreneur follows by paying the bribe $b$. In the second timing the
sequence is reversed: first the entrepreneur pays $b$, then the bureaucrat performs $a$.

Timing 1 ($T1$) is described in Figure 1. In the first node, $B_0$, the bureaucrat has two feasible actions: doing nothing ($n$) or performing the action ($a$). If he chooses $a$, the entrepreneur moves ($E_0$). He can: denounce the bureaucrat ($d$); do nothing ($n$); or pay the bribe ($b$). If the entrepreneur chooses $d$, corruption is proved, he is subject to a reduced sanction, while the bureaucrat must face the full sanction. If he chooses $n$ or $b$ the bureaucrat has to move again ($B_1$ or $B_2$) and he can either denounce the entrepreneur ($d$) or not ($n$). At each final node the payoffs of the two players are reported with the entrepreneur’s payoff first and the bureaucrat’s payoff second.

Timing 2 ($T2$) is described in Figure 2. At the first node, $E_0$, the entrepreneur has two available actions: either to pay the bribe ($b$), or not to pay it ($n$). If he pays the bribe, the game moves to node $B_0$. At this node the bureaucrat has three available actions: denouncing the entrepreneur ($d$); doing nothing ($n$); and performing the action ($a$). If the bureaucrat chooses $n$, the game reaches node $E_1$ where the entrepreneur can either denounce the bureaucrat ($d$) or not ($n$), and ends with the correspondent payoffs. If the bureaucrat performs $a$, the game gets to node $E_2$ where the entrepreneur has the same set of actions as in $E_1$ ($d$ or $n$). The payoffs are reported at the final nodes as in Timing 1.

The game under the different timings is solved by backward induction, and the equilibrium concept used is that of Subgame Perfect Nash Equilibrium (SPNE).

Given the legal framework and the parameters of the LP (if any), players can choose the timing of the illegal transaction and the size of the bribe $b$ to facilitate the illegal transaction. Therefore, in the remainder of the paper we will say that, given a leniency program, (occasional or repeated) corruption is enforceable if there is at least one timing of the one-shot corrupt transaction and one $b \in [c, v]$ such that the game has a SPNE in which the bureaucrat (always) performs the illegal action $a$, the entrepreneur (always) pays the bribe $b$, and neither player (ever) denounces the other. Conversely, we will say that corruption is not enforceable when no such equilibrium exists.

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\(^{13}\)Allowing for continuous action/bribe spaces does not affect the qualitative conclusions of the model, while it substantially complicates and lengthen the analysis of repeated transactions.
3 One-shot exchange

We begin to characterize the effects of leniency on one-shot corrupt transactions, so that the reader becomes familiar with the mechanism of the sequential games (T1 and T2). In the next section we proceed with the more complex analysis of the repeated illegal exchange.

As anticipated in the introduction, we will show here that badly designed “moderate” LPs, common in reality, may end up enforcing otherwise unfeasible occasional illegal transaction by providing the first party that delivers with a credible threat – denouncing the corrupt deal to law enforcers – that forces the other party to comply.

Consider first the benchmark case where there is no leniency, so that \( S_i = RS_i (RF_i = F_i) \) for all \( i \). Then whatever timing is chosen, the party that moves first cannot credibly threaten to report the illegal agreement to the law enforcer if the second mover does not deliver. By doing so (choosing \( d \) at he last node) he would face the full sanction incurring an additional loss. Then the second mover’s best action is \( n \), i.e. to keep the gain from the partially executed illegal transaction (the value of the investment \( v \) for the entrepreneur in T1; the bribe \( b \) for the bureaucrat in T2) without performing the action required by the illegal agreement. Knowing this, the first mover will not enter into the illegal agreement in the first place, and corruption will not occur. Note that this reasoning does not depend on the fine \( F_i \), it applies even when \( F_i = 0 \). This proves the first, rather unsurprising result.

**Proposition 1** Absent leniency programs, one-shot illegal transactions are not enforceable.

For an occasional illegal exchange to become enforceable in at least one of the two timings three conditions must be satisfied. The first condition regards the second mover (e.g. the entrepreneur in T1) and it is a **no-reporting condition**: respecting the agreement at the second node (choosing \( b \) in T1, or \( a \) in T2) must be weakly preferred to reporting (choosing \( d \)) and cashing the possible reward. This condition is:

\[
-RF_E \leq v - b \text{ in T1},
\]

\[
-RF_B \leq b - c \text{ in T2}.
\]
The other two conditions regard the first mover (e.g. the bureaucrat in T1). The first can be called the **credible threat condition**. The reduced fine for the first mover must be such that he can credibly threaten to report to the law enforcer (to choose \( d \) at the last node down) if the other player does not respect the agreement (choosing \( n \) at the second node). This condition is:\(^{14}\)

\[-RF_B \geq 0 \text{ in } T_1,\]
\[-RF_E \geq 0 \text{ in } T_2.\]

The second condition concerning the first mover, also a no-reporting constraint, can be labelled the **credible promise condition**. The reduced fine for the first mover must be such that he can credibly promise he will not report (he will not choose \( d \) at the last node up) if the other player obeys to the agreement at the second node. This condition is:

\[-RF_B \leq b \text{ in } T_1,\]
\[-RF_E \leq v \text{ in } T_2.\]

Since these three conditions must be simultaneously satisfied, the illegal transaction is enforceable in T1 if and only if

\[-RF_E \leq v - b, \text{ and } 0 \leq -RF_B \leq b;\]

and it is enforceable in T2 if and only if

\[-RF_B \leq b - c, \text{ and } 0 \leq -RF_E \leq v.\]

Consider now the effects of agents’ ability to optimally set the bribe to facilitate the enforcement of the illegal transaction.

\(^{14}\)A strictly positive probability of detection when nobody reports would weaken this constraint by providing an additional reason to turn in one’s partner if he cheats in a one-shot deal: avoiding the risk to get caught and punished.
In $T_1$ there is a conflict between the two players. If they reduce the value of $b$ in order to make the “no-reporting condition” less stringent for the entrepreneur, they tighten the “credible promise condition” for the bureaucrat. Because of this trade-off, if $-RF_E - RF_B > v$ there is no $b$ that allows the two players to satisfy both conditions (combining the two constraints we obtain $-RF_E - RF_B \leq v$). If on the other hand $-RF_E - RF_B \leq v$, there is always some $b \in [c, v]$ such that $-RF_E \leq v - b$ and $-RF_B \leq b$, unless $-RF_E > v - c$, or $-RF_B > v$. This reasoning is summarized by the following Lemma.

**Lemma 1** A one-shot illegal transaction that follows $T_1$ is enforceable for some $b \in [c, v]$ if and only if the following conditions are simultaneously satisfied:

\[
-RF_E - RF_B \leq v; \\
-RF_E \leq v - c; \\
0 \leq -RF_B \leq v.
\]

In $T_2$ there is not such a tension between conditions and, if necessary, agents can push the bribe up to $b = v$ in order to make the illegal exchange enforceable. The necessary and sufficient conditions that must be satisfied for a corrupt exchange being enforceable in $T_2$ are defined in the following Lemma.

**Lemma 2** A one-shot illegal transaction that follows $T_2$ is enforceable for some $b \in [c, v]$ if and only if the following conditions are simultaneously satisfied:

\[
-RF_B \leq v - c; \\
0 \leq -RF_E \leq v.
\]

Consider now the effects of agents’ ability to chose the timing of the transaction. Since players can choose the order in which they move after the government has chosen the LP, corruption is enforceable if it is so for some $b$ in at least in one of the two timings. Therefore, we can state this section’s main result.
Proposition 2 *Leniency programs satisfying the conditions in Lemma 1, those in Lemma 2, or both enforce one-shot sequential illegal transactions.*

These results are summarized by Figure 3, where the white area is formed by counterproductive LPs that render one-shot transactions enforceable, and the grey area is formed by LPs that do not.

4 Repeated exchange

Proposition 2 already tells us that badly designed LPs can greatly facilitate the enforcement of long-term illegal transactions, besides enforcing occasional ones. Since the repeated play of the Nash equilibrium of the stage game is always an equilibrium of the correspondent repeated game (there are no gains from deviating from a Nash equilibrium), when LPs make one-shot transactions enforceable they also make the illegal trade relation consisting of the repetition of such transactions enforceable.

Corollary 1 *Leniency programs satisfying the conditions in Lemma 1, those in Lemma 2, or both make also repeated sequential illegal transaction enforceable, independent of how often they are repeated and of the level of the discount factor:*

Though, there is more going on for repeated illegal transactions when LPs are introduced. So in this section we characterize the effects of all feasible LPs on infinitely repeated illegal relations (and on finitely repeated relations with uncertain end) that have the game described in the previous sections – under the alternative timings $T1$ or $T2$ – as their stage game.

4.1 Benchmark: no leniency

As usual in the implicit/relational contracts literature (e.g. MacLeod and Malcomson, 1989; Carmichael, 1989), we assume that agents sustain the illegal relation by the threat of termination: if a party deviates once, the other party abandons the corrupt relation and behaves according to his/her static best response function (reporting, when it is profitable
to do so) forever after.\textsuperscript{15}

Let us consider first the benchmark situation where there is no leniency. If the stage-game is played according to $T_1$, each period the bureaucrat performs the action first and the entrepreneur can then deviate unilaterally at node $E_0$ by not paying the promised bribe $b$ (since $RF_i = F_i > 0$, at $B_1$ the bureaucrat chooses $n$). Since the entrepreneur’s per-period gains from the corrupt relation are $v - b$, he will stick to his promise and pay the bribe as long as

$$b \leq \frac{\delta}{1 - \delta}(v - b), \quad \Rightarrow \quad \delta \geq \delta^{T_1} = \frac{b}{v}. \quad \text{(A)}$$

The bureaucrat would then not deviate at $B_2$ (he would lose strictly by choosing $d$); therefore, as long as condition (A) is satisfied, the repeated corrupt exchange relation can be supported in SPNE under Timing 1.

If the stage-game is played according to $T_2$, each period the entrepreneur pays the bribe first, therefore the bureaucrat has the possibility to deviate unilaterally at node $B_0$ by not providing the favor thereby saving the cost $c$ (since $RF_i = F_i > 0$, at $E_1$ the entrepreneur chooses $n$). Given that the bureaucrat’s per-period gains from the corrupt relation are $b - c$, the bureaucrat will stick to his promise to perform the action $a$ as long as

$$c \leq \frac{\delta}{1 - \delta}(b - c), \quad \Rightarrow \quad \delta \geq \delta^{T_2} = \frac{c}{b}. \quad \text{(B)}$$

and since the entrepreneur would lose strictly by deviating at node $E_2$, any repeated corrupt exchange relation that satisfies condition (B) can be supported in SPNE under Timing 2.

However, agents can optimally choose the timing of the stage game and the size of the bribe to satisfy the incentive constraints (A) or (B). It is immediate to verify that setting $b = c$ (leaving all gains from corrupt trade to the entrepreneur) relaxes condition (A) as far as possible, making the corrupt relation supportable under Timing 1 at the lowest discount factor $\delta^{T_1} = \min_{b, c \in [v, v]} \{ \delta^{T_1} = \frac{b}{v} \} = \frac{v}{b}$. Analogously, setting $b = v$ (leaving all gains from corrupt trade to the bureaucrat) relaxes condition (B) as far as feasible,

\textsuperscript{15}Considering alternative threats, weaker (e.g. interrupting the relation for a finite number of periods) or stronger (e.g. violent revenge), would substantially complicate the model, affect parameter values, but would not affect our qualitative results.
making the corrupt relation supportable under Timing 2 at the lowest discount factor $\delta^{T2} = \min_{b \in [c,v]} \{\delta^{T2} = \frac{c}{b}\} = \frac{c}{v}$.

Since $\delta^{T1} = \delta^{T2} = \frac{c}{v}$ we can state the following benchmark result.

**Proposition 3** Absent leniency programs, a repeated illegal transaction is sustainable in SPNE if and only if $\delta \geq \delta^* = \frac{c}{v}$.

It is worth noting that the “credible threat condition” defined in the previous section may not be relevant in a repeated game: if $\delta > \delta^*$, the perspective of losing gains from future corrupt deals when cheating is sufficient to induce the second mover to keep his promises.

### 4.2 Leniency programs and repeated exchange: Instruments

Characterizing the effects of all feasible LPs on repeated illegal relations when parties can endogenously choose timing and distribution of illegal gains to facilitate enforcement is difficult. To simplify exposition and shorten proofs we introduce an instrumental variable and a definition. These are first clarified for Timing 1, and then used more succinctly for Timing 2.

We let $I_{ih}(l,b)$ denote the “incentive to deviate” for player $i = E, B$, in Timing $h = 1, 2$, defined as the short-run payoff player $i$ gets by choosing an optimal defection (which defection is optimal depends on the LP and the value of $b$ chosen by the two players), divided by the short-run payoff he gets if the transaction goes through (the per-period payoff from the illegal relation, which also depends on $b$). For instance, if $-RF_B > 0$, the optimal defection from the corrupt agreement for the entrepreneur in Timing 1 is to denounce the bureaucrat at node $E_0$, as the other possibilities (doing nothing, $n$) would induce the bureaucrat to denounce the entrepreneur. In this case the incentive to deviate for the entrepreneur is $I_{E1}(b) = -RF_E/(v - b)$.

We can now introduce the definition.

**Definition 1** Given a leniency program $l$, we say that a player $i$ cannot have an incentive to deviate in $T_h$ if, for any $b \in [c,v]$, $I_{ih}(l,b) \leq 1$. 
These building blocks can be used to identify parties optimal choice of $b$, hence the crucial incentive compatibility constraint (incentive to defect) under each Timing, for any LP.

4.2.1 Timing 1

The bureaucrat may deviate in Timing 1 only by choosing $d$ at node $B_2$. Therefore, given any LP, it is $I_{B1}(l, b) = \frac{c - RF_B}{b - c}$, as the bureaucrat’s optimal defection is independent of $l$.

For the entrepreneur the optimal defection in Timing 1 depends on $l$ as he may deviate by choosing either $d$ or $n$ at node $E_0$. If $-RF_B \geq 0$, his optimal defection consists in choosing $d$ at node $E_0$, whatever is the value of $-RF_E$. If $-RF_B < 0$ his optimal defection is $n$ if $-RF_E \leq v$ and becomes $d$ if $-RF_E > v$. Therefore we have:

$$I_{E1}(l, b) = \begin{cases} \frac{RF_B}{v-b} & \text{if } -RF_B \geq 0, \text{ or if } -RF_B < 0 \text{ and } -RF_E > v \\ \frac{1}{v-b} & \text{if } -RF_B < 0 \text{ and } -RF_E \leq v \end{cases}.$$

We are now ready to concisely determine parties’ optimal bribe $b^*_1(l)$, the one at which the repeated illegal transaction can be supported in SPNE at the lowest discount factor under Timing 1 and leniency program $l$.\footnote{Hence $b^*(l)$ is the level of $b$ that minimizes the deterrence effect of $l$. The notion of optimality employed here refers only to the consequences of $b$ on the enforceability of the illegal relation, it does not imply that parties will always set $b = b^*$. If different values of $b$ suffice to avoid the deterrence effects of the LP (if parties’ incentive-compatibility condition is not binding given $l$ and the discount factor), the parties choice of $b$ may depend on other factors, such as parties’ bargaining power, and differ from $b^*(l)$.}

For any LP such that $-RF_B \leq c$ it is $I_{1E}(-RF_B \leq c, b) \leq 0$ for any $b \in [c, v]$, so the bureaucrat cannot have an incentive to deviate, hence $b^*_1(-RF_B \leq c)$ is the level of $b$ that minimizes the entrepreneur’s incentive to deviate\footnote{We construct $b^*(l)$ as a function, ignoring that when $l$ is such that both parties cannot have an incentive to deviate any value of $b$ is “optimal” in the sense discussed above. This simplifies exposition and proofs without any loss of generality.}, i.e.

$$b^*_1(-RF_B \leq c) = \arg \min_b I_{1E}(-RF_B \leq c, b) = c.$$

If $-RF_B > c$ and $-RF_E \leq 0$, the entrepreneur cannot have an incentive to deviate and
\[ b_1^*(-RF_B > c; -RF_E \leq 0) = v \] since this minimizes the bureaucrat’s incentive to deviate \[ I_{1B}(-RF_B > c; -RF_E \leq 0; b). \]

If \(-RF_B > c\) and \(-RF_E > 0\), call it \(l'\), the LP is such that for some \(b \in [c, v]\) we have both \(I_{B1} > 1\) and \(I_{E1} > 1\); in this case parties choose \(b_1^*(l')\) so that,

\[ b_1^*(l') = \arg \min_b \left( \max \{ I_{E1}(l', b), I_{B1}(l', b) \} \right), \]

and since \(\frac{\partial I_{1E}}{\partial b} < 0\) and \(\frac{\partial I_{1B}}{\partial b} > 0\), \(b_1^*(l')\) must be such that \(I_{E1}(l', b^*) = I_{B1}(l', b^*)\). Solving this equality for \(b\), we obtain:

\[ b_1^*(l') = \frac{vRF_B + cRF_E + cv}{RF_B + RF_E + c} = \tilde{b}_1. \]

Summarizing, for Timing 1 we have:

\[ b_1^*(l) = \begin{cases} 
  c & \text{if } l : -RF_B \leq c \\
  v & \text{if } l : -RF_B > c \text{ and } -RF_E \leq 0 . \\
  \tilde{b}_1 & \text{if } l : -RF_B > c \text{ and } -RF_E > 0 
\end{cases} \]

Knowing \(b_1^*(l)\), we can identify the party with the highest incentive to deviate (if any) under T1 and different LPs, and define the function

\[ I_1(l) = \max \{ I_{E1}(l, b_1^*(l)), I_{B1}(l, b_1^*(l)), 1 \}, \]

mapping the set of all LPs on the set of real numbers. The function \(I_1(l)\) defines the highest incentive to deviate from the illegal agreement under Timing 1 and leniency program \(l\), given that parties choose \(b = b_1^*(l)\) to minimize such incentive.\(^{18}\)

### 4.2.2 Timing 2

We have

\[ I_{E2}(l, b) = \frac{-b - RF_E}{v - b}, \quad \text{and} \]

\[ I_{B2}(l, b) = \begin{cases} 
  \frac{-RF_B}{b - c} & \text{if } -RF_E \geq 0, \text{ or if } -RF_E < 0 \text{ and } -RF_B > b \\
  \frac{b - c}{b - c} & \text{if } -RF_B < 0 \text{ and } -RF_E \leq b 
\end{cases} . \]

\(^{18}\)The minimum value of \(I_1(l)\) is set equal to 1 because if both \(I_{E1}\) and \(I_{B1}\) are smaller than 1 it does not matter which one is the highest as neither player can have an incentive to deviate.
If \(-RF_E \leq v\), the entrepreneur \textit{cannot} have an incentive to deviate as \(I_{E2}(-RF_E \leq v, b) \leq 1\) for any \(b \in [c, v]\), hence parties set \(b^*_2(-RF_E \leq v) = v\) to minimize \(I_{B2}(l, b)\).

If \(-RF_E > v\) and \(-RF_B \leq 0\) the bureaucrat \textit{cannot} have an incentive to deviate, hence \(b^*_2(-RF_E > v; -RF_B \leq 0) = c\) to minimize \(I_{E2}(l, b)\).

If \(-RF_E > v\) and \(-RF_B > 0\), call this LP \(l''\), the LP is such that \(I_{E2} > 1\) and \(I_{B2} > 1\) for some \(b \in [c, v]\), hence
\[
b^*_2(l'') = \arg \min \left( \max \{ I_{E2}(l'', b), I_{B2}(l'', b) \} \right).
\]
Again \(b^*_2(l'')\) must be such that \(I_{E2}(l'', b^*_2) = I_{B2}(l'', b^*_2)\), and solving the equality for \(b\), we obtain
\[
b^*_2(l'') = \frac{1}{2} \left( c - RF_E - RF_B + \sqrt{RF_E^2 + 2cRF_E + 2RF_E RF_B + c^2 - 2cRF_B + RF_B^2 + 4vRF_B} \right) \equiv \hat{b}_2
\]
Therefore, we can write:
\[
b^*_2(l) = \begin{cases} 
v & \text{if } l : -RF_E \leq v \\
c & \text{if } l : -RF_E > v \text{ and } -RF_B \leq 0 \\
\hat{b}_2 & \text{if } l : -RF_E > v \text{ and } -RF_B > 0
\end{cases}.
\]
Again, we can now define the relation between the LP and the strongest incentive for Timing 2, \(I_2(l)\), where
\[
I_2(l) = \max \{ I_{E2}(l, b^*(l)), I_{B2}(l, b^*(l)), 1 \}.
\]

4.2.3 No leniency

Finally, when there are no LPs the incentive to deviate for the entrepreneur in \(T1\) and for the bureaucrat in \(T2\), under the optimal choice of \(b^*_1 = c\) and \(b^*_2 = v\), is the same \(I = v/(v-c)\), which defines a useful benchmark.

4.3 Leniency programs and repeated exchange: Results

It is now time to harvest the fruits of the investment in notation and definitions in the previous section. We first state an immediate but very useful lemma.\(^{19}\)

\(^{19}\)For the sake of crispness we will sometimes write that a LP “facilitates” (or “hinders”) an illegal relation, meaning that the LP relaxes (or tightens) parties’ incentive compatibility constraints, making the repeated
Lemma 3 A leniency program $l$, (a) facilitates, (b) does not affect, (c) hinders (the enforcement of) a repeated illegal transaction following Timing $h$ ($h = 1, 2$) if and only if:

(a) $I_h(l) < I$;
(b) $I_h(l) = I$;
(c) $I_h(l) > I$.

Proof. Given $l$, player $i = E, B$ prefers to defect in $Th$, if and only if $I_{ih}(l) > I$. Hence Lemma 3 applies. If condition (i) holds we have $b^*(l) = c$, as $-RF_B \leq c$; therefore $I_{E1} = \frac{-RF_E}{v-c}$ and $I_1 = \max \{ I_{E1}, 1 \}$, and $I_1 < I$ as $-RF_E < v$. If condition (ii) is satisfied, then $b^*(l) = v$ as $-RF_B > c$ and $-RF_E \leq 0$. Hence, $I_{B1} = \frac{-c-RF_B}{v-c}$ and $I_1 = \max \{ I_{B1}, 1 \}$, and $I_1 < I$ as $-RF_B < v+c$. If the LP satisfies condition (iii), $b^*(l) = \hat{b}_1$, and $I_{E1} = \frac{-RF_E}{v-b_1} = \frac{-c-RF_b}{h-c} = I_{B1}$ by definition. We can write $I_1 = I_{E1} = \frac{-RF_b-RF_E-c}{v-c} < I$ as $-RF_E - RF_B < v+c$. The proof that the same conditions are necessary is the appendix. ■

Lemma 4 When the stage game follows Timing 1 a LP facilitates a repeated illegal transactions if and only if one of the following conditions is satisfied:

(i) $-RF_E < v$, and $0 \leq -RF_B \leq c$;
(ii) $-RF_E \leq 0$, and $c < -RF_B < v+c$;
(iii) $-RF_E > 0$ and $-RF_B > c$, and $-RF_E - RF_B < v+c$.

Proof. We prove that these conditions are sufficient by showing that if one of them is satisfied $I_1 < I$ hence Lemma 3 applies. If condition (i) holds we have $b^*(l) = c$, as $-RF_B \leq c$; therefore $I_{E1} = \frac{-RF_E}{v-c}$ and $I_1 = \max \{ I_{E1}, 1 \}$, and $I_1 < I$ as $-RF_E < v$. If condition (ii) is satisfied, then $b^*(l) = v$ as $-RF_B > c$ and $-RF_E \leq 0$. Hence, $I_{B1} = \frac{-c-RF_B}{v-c}$ and $I_1 = \max \{ I_{B1}, 1 \}$, and $I_1 < I$ as $-RF_B < v+c$. If the LP satisfies condition (iii), $b^*(l) = \hat{b}_1$, and $I_{E1} = \frac{-RF_E}{v-b_1} = \frac{-c-RF_b}{h-c} = I_{B1}$ by definition. We can write $I_1 = I_{E1} = \frac{-RF_b-RF_E-c}{v-c} < I$ as $-RF_E - RF_B < v+c$. The proof that the same conditions are necessary is the appendix. ■

illegal transaction enforceable (sustainable in SPNE) at a lower (or higher) minimum discount factor.
Lemma 5 When the stage game follows Timing 2 a LP facilitates a repeated illegal transactions if and only if one of the following conditions is satisfied:

(i) \( 0 \leq -RF_E \leq v, \) and \( -RF_B < v; \)

(ii) \( 0 \leq -RF_E < v + c, \) and \( -RF_B \leq 0; \)

(iii) \( -RF_E > v \) and \( -RF_B > 0, \) and \( -cRF_B - vRF_E < v(v + c). \)

Proof. If condition (i) holds, we have \( b^*(l) = v \) as \( -RF_E \leq v; \) therefore \( I_{B2} = \frac{-RF_E}{v-c} \) and \( I_2 = \max \{ I_{B2}, 1 \} \), and \( I_2 < I \) as \( -RF_B < v. \) If condition (ii) is satisfied, then \( b^*(l) = c \) as \( -RF_E > v \) and \( -RF_B \leq 0; \) Hence \( I_{E2} = \frac{-c}{v-c} -RF_E, \) \( I_2 = \max \{ I_{E2}, 1 \} \), and \( I_2 < I \) as \( -RF_E < v + c. \) If the LP satisfies condition (iii), we get \( b^*(l) = \tilde{b}_2, \) and \( I_{E2} = I_{B2} = I_2 \) by definition. By substituting \( \tilde{b}_2 \) we obtain \( I_2 < I \) as \( -cRF_B - vRF_E < v(v + c). \) The proof that the same conditions are necessary is the appendix.

By imposing the more stringent condition that the relevant player cannot have an incentive to deviate \( (I_h = 1), \) one obtains the sets of LPs that make the repeated illegal exchange (under each timing) enforceable at any discount factor, i.e. the sets defined by Lemma 1 and Lemma 2.

Consider now the effects of agents’ ability to choose the timing of the stage game. Since the repeated illegal transaction is feasible when it is enforceable under at least one timing, LPs are counterproductive when they facilitate the enforcement of the transaction in at least one timing, while they have deterrence effects only when they make the enforcement harder under both timings. Taking this into account, and combining the previous Lemmas, we obtain the set of LPs that facilitate long-term illegal transactions.

Proposition 4 Leniency programs satisfying one of the conditions in Lemma 4, in Lemma 5, or in both facilitate repeated illegal transactions.

5 Complete characterization and policy implications

To present concisely our analytical conclusions we partition the set of feasible LPs, \( L, \) in the 4 subsets \( L_1 - L_4 \) described in Figure 4. Their formal definition is provided in the Appendix,
along with the proof of the following Theorem.

**Theorem 1**

i) *Leniency programs belonging to $L_1$ are irrelevant:* they do not affect one-shot illegal transactions nor repeated ones.

ii) *Leniency programs belonging to $L_2$ are highly counterproductive:* they enforce (otherwise unenforceable) one-shot illegal transactions, and make repeated ones sustainable at any discount factor.

iii) *Leniency programs belonging to $L_3$ are counterproductive:* they do not affect one-shot illegal transactions, but they make repeated ones sustainable at lower discount factors.

iv) *Leniency programs belonging to $L_4$ are effective:* they do not affect one-shot illegal transactions, and they raise the minimum discount factor at which repeated ones are sustainable.

Looking at Figure 4 should be sufficient to raise concerns about how leniency for spontaneously self-reporting parties is provided in reality. The formal or informal leniency programs implemented by law enforcing agencies around the world appear to fall, in most cases, within sets $L_1 - L_3$.

At this point, the following remark is in order.

**Remark 1** *Allowing parties to choose other timings besides T1 and T2 (weakly) enlarges the set of counterproductive LPs.*

Allowing, for example, part of the bribe to be paid before the bureaucrat undertakes the action and part after, can only reinforce the counterproductive effects of LPs. This is because, both in the one-shot and in the repeated case, the transaction is enforceable as long as it is so under at least one timing of the stage game. Therefore, adding additional timings to T1 and T2 among which agents can choose cannot affect the validity of conclusions ii) and iii) of Theorem 1. Additional timings, however, may change conclusions i) or iv) of the Theorem by ensuring that some LPs in the sets $L_1$ or $L_4$ facilitate the enforcement of illegal transactions following one of these timings. That is, our characterization of counterproductive LPs should be regarded as a prudential one.
The following corollary to Theorem 1 (hence limited to T1 and T2) confirms Spagnolo’s (2000) results on the potential deterrence effects of “high powered” leniency programs.

**Corollary 2** Given parties’ discount factor $\delta^p \in [0, 1)$, if $\min \{S_B; S_E\} \geq \max \{-RS_B; -RS_E\}$ and at least one of the following conditions is satisfied leniency is highly effective, as it costlessly deters the illegal transaction:

\begin{align*}
i) \quad -RF_B &> \frac{v - \delta^p c}{1 - \delta^p}; \\
ii) \quad -RF_E &> \frac{v - \delta^p c}{1 - \delta^p}; \\
iii) \quad -RF_E &> \frac{v - c}{1 - \delta^p} \text{ and } -RF_B > \frac{v - c}{1 - \delta^p} \text{; or} \\
iv) \quad -RF_E &\geq v \text{ and } -RF_B \geq c \text{ and } -RF_E - \delta^p RF_B > \frac{v - \delta^p c}{1 - \delta^p}. 
\end{align*}

Contrary to what one usually obtain with single wrongdoers and isolated/individual crime, when leniency is available complete deterrence of illegal transactions may be optimal, since it can be achieved at zero cost. Sufficiently high rewards for illegal traders that turn in their partners may elicit wrongdoers private information and by so doing undermine trust between criminals at the point of making illegal transactions not sustainable. This achieves the first best: complete deterrence and no costs of investigation. Of course, leniency programs with rewards – as any “high powered incentive scheme” – may also be problematic and lead to distortions, hence their implementation requires great care. For example, the above corollary requires $\min \{S_B; S_E\} \geq \max \{-RS_B; -RS_E\}$ because if rewards for the self-reporting party are larger than the sanctions incurred by the other party, agents might start entering illegal transactions only to self-report and cash the rewards in turn.\textsuperscript{20} Also, we mentioned that high rewards might give agents incentives to “fabricate information,” hence precautions should be taken against this possibility; for example, by not allowing rewarded parties to testify in front of courts, and by increasing sanctions for false reporting, and for law enforcers that (sometimes) induce it.\textsuperscript{21}

\textsuperscript{20}For more detailed discussion of pros and cons of leniency programs with rewards see Spagnolo (2000) and Rey (2000).

\textsuperscript{21}Once more, we encourage readers to visit http://www.pbs.org/wgbh/pages/frontline/shows/snitch/, an excellent overview of how leniency can be misused by law enforcing agencies.
6 Conclusions

Within a stylized sequential bilateral exchange model, we have characterized the effects on the sustainability of illegal transactions, occasional and repeated, of all conceivable parametrizations of leniency programs for wrongdoers that spontaneously self-report, turning in their illegal partners.

The results highlight the great risk involved in implementing moderate leniency programs, as usually done in reality. Economists are aware that badly designed incentive schemes may be counterproductive, and leniency programs are incentive schemes. The form of leniency implemented around the world may be counterproductive, or highly so; it may end up generating illegal transactions, rather than deterring them.

Extending the model by introducing asymmetric information, uncertainty, violent revenge, and other important features of reality will surely modify the boundaries of our characterization, and it will be very interesting to understand in which direction. However, we are fairly confident that our qualitative conclusions will not change. Leniency for wrongdoers that spontaneously self-report is a powerful but dangerous tool, if badly used. It could in principle be effective in deterring illegal transactions, but to be so it may have to be sufficiently generous.
7 Appendix

7.1 Proof of Theorem 1

To prove Theorem 1 we will use the following Lemmas.

Lemma 6 A LP is irrelevant in $T_1$ if and only if one of the following conditions is satisfied:

(Ai) $-RF_B < 0$ and $-RF_E \leq v$;

(Aii) $0 \leq -RF_B \leq c$ and $-RF_E = v$;

(Aiii) $-RF_B = v + c$ and $-RF_E \leq 0$;

(Aiv) $-RF_B > c$ and $-RF_E > 0$ and $-RF_E - RF_B = v + c$.

Proof. If condition (Ai) holds we have $b^*(l) = c$, as $-RF_B < c$, and $I_{E1} = \frac{v}{v-c}$, as $-RF_B < 0$ and $-RF_E \leq v$; therefore, $I_1 = I_{E1} = I$. If condition (Aii) is satisfied, then $b^*(l) = c$, as $-RF_B \leq c$; $I_{E1} = \frac{-RF_B}{v-c}$, as $-RF_B \geq 0$, and $I_1 = I_{E1} = I$ as $-RF_E = v$. If condition (Aiii) is satisfied, then $b^*(l) = v$, as $-RF_B > c$ and $-RF_E \leq 0$; hence, $I_1 = I_{B1} = I$, as $-RF_B = v + c$. Finally, if the LP satisfies condition (Aiv), $b^*(l) = \tilde{b}_1$, and $I_{E1} = \frac{-RF_E}{v-b_1} = \frac{v-c}{b_1-c} = I_{B1}$ by definition; hence $I_1 = I_{E1} = \frac{-RF_B-RF_E-c}{v-c} = I$ as $-RF_E - RF_B = v + c$.

Lemma 7 A LP is irrelevant in $T_2$ if and only if one of the following conditions is satisfied:

(Bi) $-RF_E < 0$ and $-RF_B \leq v$; or

(Bii) $0 \leq -RF_E \leq v$ and $-RF_B = v$;

(Biii) $-RF_E = v + c$ and $-RF_B \leq 0$;

(Biv) $RF_E > v$ and $-RF_B > 0$ and $cRF_B - vRF_E = v(v + c)$. 

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Proof. If condition (Bi) holds we have \( b^*(l) = v \), as \( -RF_E < v \); and \( I_{B2} = \frac{v}{v-c} \), as \( -RF_E < 0 \) and \( -RF_B \leq v \); therefore, \( I_2 = I_{B2} = I \). If condition (Bii) is satisfied, then \( b^*(l) = v \), as \( -RF_E \leq v \); \( I_{B2} = \frac{-RF_B}{v-c} \) as \( -RF_E \geq 0 \); and \( I_2 = I_{B2} = I \) as \( -RF_B = v \). If condition (Biii) is satisfied, then \( b^*(l) = c \) as \( -RF_E > v \) and \( -RF_B \leq 0 \); hence \( I_2 = I_{E2} = I \) as \( -RF_E = v + c \). Finally, if condition (Biv) is satisfied, \( b^*(l) = \hat{b}_2 \), and \( I_{E2} = \frac{b-\bar{RF}_E}{v-b_2} = \frac{c-\bar{RF}_E}{b_2-c} = I_{B2} \) by definition, and plugging the value of \( \hat{b}_2 \) we get \( I_2 = I_{E2} = I_{B2} = I \) as \( -cRF_B + vRF_E = v(c+v) \).

Lemma 8 A LP is effective in \( T_1 \) if and only if one of the following conditions is satisfied:

\[
\begin{align*}
(Ci) & \quad -RF_B \leq c \quad \text{and} \quad -RF_E > v; \\
(Cii) & \quad -RF_B \geq v + c, \quad \text{and} \quad -RF_E \leq 0 \\
(Ciii) & \quad -RF_B \geq c \quad \text{and} \quad -RF_E \geq 0 \quad \text{and} \quad -RF_E - RF_B > v + c.
\end{align*}
\]

Proof. If condition (Ci) is satisfied we have \( b^*(l) = c \), as \( -RF_B \leq c \); and \( I_1 = I_{E1} = \frac{-RF_E}{v-c} > I \), as \( -RF_E > v \). If condition (Cii) holds, then \( b^*(l) = v \), as \( -RF_B > c \) and \( -RF_E \leq 0 \); hence, \( I_1 = I_{B1} = \frac{c-\bar{RF}_E}{v-c} > I \), as \( -RF_B > v + c \). Finally, if condition (Ciii) holds, \( b^*(l) = \hat{b}_1 \), and \( I_1 = I_{E1} = I_{B1} = \frac{-RF_E - RF_E - c}{v-c} > I \) as \( -RF_E - RF_B > v + c \).

Lemma 9 A LP is effective in \( T_2 \) if and only if one of the following conditions is satisfied:

\[
\begin{align*}
(Di) & \quad -RF_E \leq v \quad \text{and} \quad -RF_B > v; \quad \text{or} \\
(Dii) & \quad -RF_E \geq v + c \quad \text{and} \quad -RF_B \leq 0 \\
(Diii) & \quad -RF_E \geq v \quad \text{and} \quad -RF_B \geq 0 \quad \text{and} \quad -cRF_B - vRF_E > v(c+v).
\end{align*}
\]

Proof. If condition (Di) holds we have \( b^*(l) = v \), as \( -RF_E \leq v \); and \( I_{B2} = \frac{-RF_E}{v-c} > I \), as \( -RF_B > v \). If condition (Dii) is satisfied, then \( b^*(l) = c \), as \( -RF_E > v \) and \( -RF_B \leq 0 \); hence, \( I_2 = I_{E2} = \frac{c-\bar{RF}_E}{v-c} > I \), as \( -RF_E > v + c \). Finally, if condition (Diii) is satisfied, \( b^*(l) = \hat{b}_2 \), and \( I_{E2} = \frac{b-\bar{RF}_E}{v-b_2} = \frac{c-\bar{RF}_E}{b_2-c} = I_{B2} \) by definition. Plugging the value of \( \hat{b}_2 \) in this formula, we get \( I_2 = I_{E2} = I_{B2} > I \) as \( -cRF_B - vRF_E > v(c+v) \).
Proof of necessity for Lemmas 4-9

If we draw the sets defined by the conditions stated in Lemmas 4, 6 and 8 (or Lemmas 5, 7 and 9) we can easily check that all their intersections are empty and that their union forms the set of all possible LPs. This suffices to prove the “only if” part of all lemmas. ■

Proof of Theorem 1

Using Lemmas 1-9 we can define the four sets:

i) The set $L_1$ is formed by all LPs that satisfy at least one of the following conditions:

\begin{align*}
(L_1.1) & \quad -RF_E < 0 \text{ and } -RF_B < 0; \\
(L_1.2) & \quad -RF_E = c \text{ and } -RF_B = v; \\
(L_1.3) & \quad -RF_E < 0 \text{ and } -RF_B = v + c; \\
(L_1.4) & \quad -RF_E = v + c \text{ and } -RF_B < 0; \\
(L_1.5) & \quad v < -RF_E \leq v + c \text{ and } 0 < -RF_B \leq v \text{ and } \\
& \quad -cRF_B - vRF_B = v(v + c); \\
(L_1.6) & \quad 0 \leq -RF_E \leq c \text{ and } v \leq -RF_B \leq v + c \text{ and } \\
& \quad -RF_E - RF_B = v + c; \\
(L_1.7) & \quad \max \{v - c, c\} < -RF_E < v \text{ and } -RF_B = v.
\end{align*}

If condition $(L_1.1)$ is satisfied, the LP satisfies both condition $(Ai)$ of Lemma 6 and condition $(Bi)$ of Lemma 7. Hence, the LP is irrelevant in both timings. Similarly, if condition $(L_1.2)$ is satisfied, the LP satisfies both condition $(Aii)$ of Lemma 6 and condition $(Bii)$ of Lemma 7. All the other conditions defined in the Lemmas 6 and 7 are mutually incompatible. Therefore there are no other LPs that are irrelevant in both timings. However, if condition $(L_1.3)$ holds, the LP satisfies condition $(Aiii)$ of Lemma 6 and condition $(Di)$ of Lemma 9. Thus these LPs make the illegal exchange harder to enforce in $T_2$ but are irrelevant for $T_1$. As the two players can choose the timing as they wish, these LPs must
be considered irrelevant. Similarly if \((L_{1.4})\) holds the LP satisfies both condition (Biii) of Lemma 7 and condition (Ci) of Lemma 8. If \((L_{1.5})\) holds the LP satisfies conditions (Biv) of Lemma 7 and either (Ci) of Lemma 8 (if \(-RF_B \leq c\)) or (Ciii) of the same Lemma. If condition \((L_{1.6})\) is satisfied then the LP satisfies conditions (Aiv) of Lemma 6 and (Di) of Lemma 9. Finally, a LP that satisfies condition \((L_{1.7})\) also satisfies condition (Bii) of Lemma 7 and condition (Ciii) of Lemma 8.

\(\text{ii)}\) The set \(L_2\) is formed by all LPs defined by Proposition 2, from which the statement follows.

\(\text{iii)}\) The set \(L_3\) is formed by all LPs that satisfy at least one of the following conditions:

\[
\begin{align*}
(L_{3.1}) \quad &v < -RF_E \leq v + c \text{ and } 0 \leq -RF_B < v \text{ and } \\
& -cRF_B - vRF_E < v(v + c);
\end{align*}
\]

\[
\begin{align*}
(L_{3.2}) \quad &-RF_E \leq c \text{ and } v < -RF_B \leq v + c \text{ and } \\
& -RF_E - RF_B < v + c;
\end{align*}
\]

\[
\begin{align*}
(L_{3.3}) \quad &0 < -RF_E < v \text{ and } \max\{v - c, c\} < -RF_B < v \text{ and } \\
& v < -RF_E - RF_B < v + c.
\end{align*}
\]

If \(v - c < c\) we have to add the following condition:

\[
(L_{3.4}) \quad v - c < -RF_E < v \text{ and } v - c < -RF_B < c.
\]

LPs that satisfy condition \((L_{3.1})\) also satisfy condition (iii) of Lemma 5 and either condition (Cii) of Lemma 7 (if \(-RF_B \leq c\)) or condition (Ciii) of the same Lemma; hence they make the illegal exchange easier to be enforced in Timing 1 and more difficult in Timing 2. The first effect prevails as the two players are free to choose the timing of the game. In addition to this, these LPs violate the second condition of both Lemma 1 and Lemma 2 and therefore do not affect occasional illegal transactions. If condition \((L_{3.2})\) is satisfied, the LP satisfies either condition (iii) of Lemma 4 (if \(-RF_B > 0\)) or condition (ii) of the same Lemma; it also satisfies condition (Di) of Lemma 9, but violates the third condition of Lemma 1 and
the second condition of Lemma 2. Hence, the LP facilitates repeated illegal transactions that follow Timing 2, makes more difficult to enforce repeated illegal deals in Timing 1 and does not affect one-shot games. LPs that satisfy condition \( (L_3.3) \) also satisfy condition \((iii)\) of Lemma 4 and condition \((i)\) of Lemma 5 whereas they violate the first condition of both Lemma 1 and Lemma 2; therefore they facilitate repeated illegal transaction but do not affect one-shot games. If condition \( (L_3.4) \) holds, then the LP satisfies conditions \((i)\) of Lemma 4 and Lemma 5, but violates the second condition of Lemma 1 and the first condition of Lemma 2.

\( iv \) Finally, the set \( L_4 \) is formed by all LPs that satisfy at least one of the following conditions:

\[
(L_4.1) \quad -RF_B > v + c ;
\]

\[
(L_4.2) \quad -RF_E > v + c ;
\]

\[
(L_4.3) \quad -RF_E > v \text{ and } -RF_B > v ;
\]

\[
(L_4.4) \quad -RF_E \geq v \text{ and } -RF_B \geq 0 \text{ and } -vRF_E - cRF_B > v(v+c) ;
\]

\[
(L_4.5) \quad 0 \leq -RF_E \leq v \text{ and } v < -RF_B \leq v + c \text{ and } -RF_E - RF_B > v + c .
\]

If \(-RF_B > v + c\) it is immediate to verify that either condition \((C_{ii})\) or condition \((C_{iii})\) in Lemma 8 is satisfied and that either condition \((D_{i})\) or condition \((D_{iii})\) in Lemma 9 is satisfied. Therefore, the enforcement of the illegal exchange is more difficult in both timings. Similarly if \(-RF_E > v + c\) either condition \((D_{iii})\) or condition \((D_{ii})\) in Lemma 9 is satisfied and either condition \((C_{i})\) or condition \((C_{ii})\) in Lemma 8 holds. If \(-RF_B > v\) and \(-RF_E > v\) this satisfies both \((C_{ii})\) and \((D_{iii})\). If condition \((L_4.4)\) holds, the LP satisfies condition \((D_{iii})\) of Lemma 9 and either condition \((C_{i})\) of Lemma 8 (if \(-RF_B < c\)) or condition \((C_{iii})\) of the same Lemma. If condition \((L_4.5)\) is satisfied the LP satisfies conditions \((C_{iii})\) of Lemma 8 and \((D_{i})\) of Lemma 9. Also in this case the graphical construction of these four sets shows that all their intersections are empty and that their union forms the set of all possible LPs.
Proof of Corollary 1

The condition \( \min \{ S_B; S_E \} \geq \max \{ -RS_B; -RS_E \} \) ensures that the fines one party faces when the other self-report are sufficiently high that no equilibria exist where parties arrange for illegal transactions with the purpose of sharing the reward from self-reporting.

If condition i) holds, the bureaucrat has an incentive to deviate in both timings that cannot be matched by the gains he would get from an infinitely repeated illegal game. Indeed, in Timing 1, by choosing action \( d \) at node \( B_2 \) he gets \( c - RF_B \) and we have

\[
c - RF_B > \frac{b - c}{1 - \delta}
\]

for any \( \delta \) and any feasible value of \( b \), since \( -RF_B > \frac{b - c}{1 - \delta} \). In Timing 2, by choosing \( d \) at node \( B_0 \), the bureaucrat gains \( -RF_B \) and a fortiori we have

\[
-RF_B > \frac{b - c}{1 - \delta}
\]

for any \( \delta \in [0, \delta^p] \). Similar and straightforward calculations apply for conditions ii) and iii). Condition ii) assures that the entrepreneur deviates at node \( E_0 \) in Timing 1 or at node \( E_2 \) in Timing 2. If condition iii) is satisfied then the entrepreneur deviates in Timing 1 by choosing \( d \) at node \( E_0 \) and the bureaucrat defects in Timing 2 at node \( B_0 \). Finally, if the first two inequalities stated in condition iv) hold, both players may deviate in both timings, therefore after selecting one of the two timings they have to set \( b \). This will be done optimally as shown in section 4. Hence we have \( b = \hat{b}_1 \) if they play the game following Timing 1 or \( b = \hat{b}_2 \) if they prefer Timing 2. In both cases, however, neither the entrepreneur nor the bureaucrat can be compensated in to match the immediate gain they would get by defecting: in Timing 2 if the last part of condition iv) is satisfied, for any feasible value of \( b \) either the bureaucrat defects at node \( B_0 \), or the entrepreneur defects at node \( E_2 \); the same condition is sufficient, although not necessary, to obtain the same result in Timing 1. ■
References


Figure 1: Timing 1

Figure 2: Timing 2
Figure 3: Counteproductive LPs for occasional illegal transactions (white area)

Figure 4: The consequences on deterrence of LPs